THEORY OF THE INITIAL STAGE OF AN ELECTRICAL DISCHARGE IN WATER

A. I. Ioffe

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 7, No. 6, pp. 69-72, 1966

Experiments on an electrical discharge in water show that quantities such as the electrical current, voltage, channel radius, pressure in the compression pulse, and certain other quantities, as well as the variation of these quantities with time, can be determined from four given parameters, namely, the initial voltage V_0 across the capacitor, the inductance L of the discharge circuit, the capacitance C of the discharging capacitor, and the length l of the gap between the electrodes.

To describe the behavior of the discharge in time, one could try to set up a set of equations which would also include quantities characterizing the discharge circuit as well as quantities referring to the discharge channel formed as a result of breakdown. The solution of the system should then give the time dependence of the various quantities in which we are interested.

We shall suppose that the discharge occurs in an ordinary oscillatory circuit with given L and C, and that the circuit resistance is completely determined by the resistance of the discharge channel, which is a function of time. The discharge channel is a cylinder whose radius increases with time.

For the electrical circuit we can use the usual differential equation for the oscillatory circuit:

$$\frac{d^2V}{dt^2}+\frac{R_e}{L}\frac{dV}{dt}+\frac{V}{LC}=0 \quad \left(R_e=\frac{\rho_e(t)\,l}{S(t)}\right), \qquad (1)$$

where V is the voltage across the capacitor, R_e is the resistance of the discharge channel, ρ_e is the resistivity, and S(t) is the cross-sectional area of the discharge channel.

The temperature in the discharge channel has been estimated as lying in the range 10 000–30 000° K [1, 2], and although at such temperatures the plasma in the discharge channel is not completely ionized, we shall assume that the plasma resistance is determined only by the interaction between electrons and ions, and that ρ_e is given by [3]

$$\rho_e=\frac{\alpha\pi^{3/2}m_e^{1/2}e^2\ln\Lambda}{2(2kT)^{3/2}}=\frac{c_1\ln\Lambda}{T^{3/2}} \qquad \Lambda=\frac{3}{2e^3}\left(\frac{k^3T^3}{\pi n_e}\right)^{1/2}, \quad (2)$$

where m_e and e are the mass and charge of the electron, respectively, T is the temperature, and $\alpha = 1.8$ is a dimensionless coefficient; n_e is the electron density.

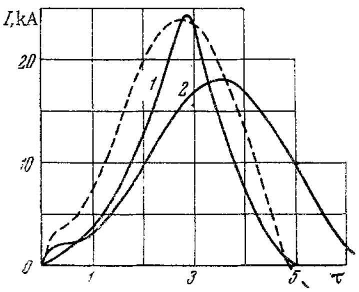

Fig. 1

If we restrict our attention to discharges in which the rate of expansion v of the channel is small in comparison with the velocity of sound c in the water, the pressure p on the channel wall is related to its rate of expansion as follows [4]:

$$p=\frac{\rho}{2\pi}\frac{d^2S}{dt^2}\ln\left(\frac{\pi^{1/2}l}{S^{1/2}}\right)-\frac{\rho}{8\pi}\left(\frac{dS}{dt}\right)^2. \qquad (3)$$

In deriving this formula it was assumed that the channel was an impermeable cylinder of length L and cross section S (ρ is the density of the fluid). The expression given by Eq. (3) is valid for $R < l$ and $l \ll cT_1$, where R is the radius of the cylinder and T_1 is the time necessary for an appreciable change in the expansion velocity.

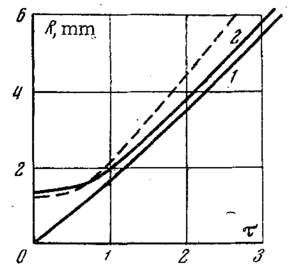

Fig. 2

Following [1], we shall assume that the power supplied the channel is expended in doing work against the surrounding medium (the work done in expanding the channel is dA = pdV) and in producing a change in the internal energy of the plasma. The plasma energy-density ε (in the pressure and temperature intervals in which we are interested) can be approximately represented by

$$\varepsilon = p/(\gamma-1), \qquad (4)$$

where p is the pressure and $\gamma = 1.2$ (the effective adiabatic exponent) [1]. The energy-balance equation can then be written in the form (per unit channel length)

$$p\frac{dS}{dt}+\frac{1}{\gamma-1}\frac{d(pS)}{dt}=C^2\left(\frac{dV}{dt}\right)^2\frac{\rho_e}{S}=$$
$$=I^2R_e \quad \left(I=C\frac{dV}{dt}\right). \qquad (5)$$

The right-hand side of this equation represents the usual Joule losses.

It can be assumed that the plasma in the discharge channel can be described by the equation of state for an ideal gas

$$p=nkT, \qquad (6)$$

where n is the particle density. In principle, Eqs. (1), (4), (6), and (7) are sufficient to describe the discharge, provided the mass of the gas in the discharge channel is constant. However, both the particle density and the mass of the gas in the channel will change as a result of evaporation of water into the channel.

We shall suppose that there is a boundary between the discharge channel and the surrounding water, and we will not take into account the transition layer between the water and channel.

Following Frenkel [5], we shall suppose that the evaporation of the liquid from the surface occurs at the rate

$$w=m\nu_0 n_0^{2/3}\exp(-u/kT_0) \quad [\text{g/cm}^2\,\text{sec}], \qquad (7)$$

where m is the mass of a molecule, ν_0 is a characteristic frequency of the order of the Debye frequency, n_0 is the particle density in the liquid, T_0 is the temperature on the surface of the evaporating liquid, and u is the evaporation energy per molecule. The evaporation is produced by the heat reaching the boundary of the liquid. We shall assume that the discharge channel behaves as a black body. Estimates based on formulas given in [6] for a temperature of about 20 000° K and a density $n = 10^{20}$ cm^{-3} yield a mean value $l \approx 0.1$ cm so that we can take it that the above assumption is valid in the first approximation. The flux of radiation from a unit area on the surface of the channel is then $q = \sigma T^4$ (the electron current is $q_e \approx \varkappa T/R$, where R is the channel radius and $\varkappa(T)$ is the electron thermal conductivity [3], which is smaller by two orders of magnitude than q for T = 20 000° K).

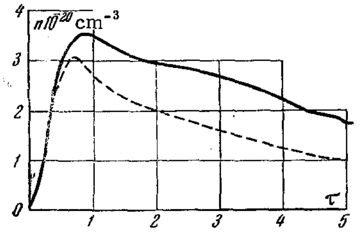

Fig. 3

We shall suppose that the flux q is used up exclusively in evaporating water into the channel. The absorption coefficient of water at $\lambda = 1500$ (the maximum of the Planck curve occurs at $T \approx 20\ 000°$ K) is about 10^4 cm^{-1}. The range of this radiation is of the order of 10^{-4} cm, and, probably most of the water will not be heated by it.

The expression given by Eq. (7) will then enable us to determine the temperature on the boundary of the liquid if we know the flux

$$q = \sigma T^4 = u n_0^{2/3} \nu_0 \exp(-u / kT_0).$$

An estimate of this kind has been carried out in [10].

For T = 20 000° K, $n_0 = 3 \cdot 10^{22}$cm^{-3}, and $\nu_0 = 10^{13}$ sec^{-1} we have $T_0 \approx 700$°K. The thermal conductivity of water is $\varkappa_1 \approx 600\ 000$ erg/ /cm·sec·deg [7] and the effective heating thickness $\delta = (\varkappa \Delta t / \rho c_p)^{1/2}$ of heated water for a time $\Delta t = 50$ µsec is about 0.0003 cm, while the flux due to heat conduction from the walls is $q_1 \approx \varkappa_1 T_0/\delta \approx 10^{11}$ erg/cm^2 sec for $q \approx 9 \cdot 10^{12}$ erg/cm^2 sec, i.e., $q_1 \ll q$. This means that we can assume that the flux of particles evaporated into the channel will be determined by q/u, where u is the evaporation energy per molecule. For the particle density n in the channel we have

$$d(nS) / dt = 2\pi R\ \sigma T^4 / u = 2\pi^{1/2} S^{1/2} T^4 / u. \qquad (8)$$

This equation closes the set of equations necessary for describing both the electrical and the channel parameters during discharge. The system consists of Eqs. (1), (4), (6), and (8) into which we must substitute the expression given by Eq. (3) for the resistance of the plasma, and eliminate temperature using the equation of state for an ideal gas, p = nkT.

The above set of equations will consist of the following: the energy-balance equation, the right-hand side of which presents the rate at which energy is introduced into the discharge channel, the equations for the particle flux, the expressions for the pressure in the channel, and the equation for the voltage across the capacitor:

$$\frac{S}{\gamma-1}\frac{dp}{dt} + \frac{p\gamma}{\gamma-1}\frac{dS}{dt} = \frac{C^2\rho_0 10^7}{S}\left(\frac{dV}{dt}\right)^2\left(\frac{p}{nk}\right)^{-3/2}$$

$$\frac{d(nS)}{dt} = \frac{2\pi^{1/2}\sigma T^4 S^{1/2}}{u},$$

$$p = \frac{\rho}{2\pi}\frac{d^2S}{dt^2}\ln\left(\frac{\pi^{1/2} l}{S^{1/2}}\right) - \frac{\rho}{8\pi}\left(\frac{dS}{dt}\right)^2,$$

$$\frac{d^2V}{dt^2} + \frac{\rho_0 l}{LS}\left(\frac{p}{nk}\right)^{-3/2}\left(\frac{dV}{dt}\right) + \frac{V}{LC} = 0. \qquad (9)$$

The above equations can be integrated under the appropriate initial conditions. Some of these are obvious: at time t = 0 the capacitor is charged to a given voltage $V(0) = V_0$ and there is no current in the circuit, which means that

$$\left.\frac{dV}{dt}\right|_{t=0} = 0 \qquad \left(I = C\frac{dV}{dt}\right).$$

The remaining initial conditions are not very definite. In [8] it was noted that the initial channel radii for the discharge gaps were 0.3–0.7 mm. If the discharge is initiated by a wire, we can assume that $S(0) = S_0$ is the cross section of the wire. For the set of equations given by (9), the initial value was chosen to be of the order of 0.1 cm^2. It was assumed that dS/dt = 0 for t = 0, and that the initial temperatures and particle density were, respectively, 10 000° K and 10^{19}cm^{-3}. At 10–30% change in T(0) and n_0 had practically no effect on the numerical solution.

The equations given by (9) were integrated numerically. They were first transformed to a dimensionless form, using the substitution

$$\tau = t / (LC)^{1/2}, \quad \xi = S / \lambda^2, \quad \beta = p / p_0,$$
$$\nu = V / V_0, \quad \mu = n / n_0,$$
$$\lambda = [C^2 V_0^2 L \cdot 5 \cdot 10^6 / l\rho]^{1/4}, \quad p_0 = [CV_0 \cdot 10^{7/2} l^3]. \qquad (10)$$

In the Eq. (9) all the mechanical quantities must be taken in the CGS system; the voltage is given in volts, the inductance in henries, and the capacitance in faradays.

In the new variables the set of equations given by (9) assumes the form

$$\frac{d\beta}{d\tau} = \frac{\omega_2}{\xi^2}\left(\frac{\mu}{\beta}\right)^{3/2}\left(\frac{d\nu}{d\tau}\right)^2 - \frac{1{,}2\beta}{\xi}\left(\frac{d\xi}{d\tau}\right),$$

$$\frac{d\mu}{d\tau} = \frac{\omega_3}{\xi^{1/2}}\left(\frac{\beta}{\mu}\right)^4 - \frac{\mu}{\xi}\frac{d\xi}{d\tau},$$

$$\frac{d^2\xi}{d\tau^2} = \frac{\omega_4\beta\xi + \omega_5(d\xi/d\tau)^2}{\xi\ln(\omega'/\xi^{1/2})},$$

$$\frac{d^2\nu}{d\tau^2} = -\nu - \frac{\omega_1}{\xi}\left(\frac{\mu}{\beta}\right)^{3/2}\left(\frac{d\nu}{d\tau}\right),$$

$$\omega_1 = \frac{(LC)^{1/2}(n_0k)^{3/2} l\rho_0}{L\lambda^2 p_0^{3/2}}, \qquad \omega_2 = \frac{\rho_0 (n_0k)^{3/2} V_0^2 10^7 C^{3/2}}{5\lambda^4 p_0^{5/2} L^{1/2}},$$

$$\omega_3 = \frac{2\pi^{1/2}\sigma p_0^4 (LC)^{1/2}}{u\lambda^2 n_0^5 k^4}, \qquad \omega_4 = \frac{2\pi p_0 LC}{\rho\lambda^2},$$

$$\omega_5 = 1/8\pi, \quad \omega' = \pi^{1/2} l/\lambda. \qquad (11)$$

It was assumed in the integration that ln Λ = const, $\rho_0 = c_1 \ln\Lambda$.

The results of the integration are shown in Figs. 1–8 (curve 2). Existing experimental data (curve 1) are also indicated. The results shown in Figs. 1, 2, 3, and 4 refer to a discharge with the following parameters [1]:

$$C = 150\ \mu F, \quad L = 2\ \mu H,$$

$$V_0 = 6\text{kV}, \ l = 7\text{cm}, \ \sqrt{LC} = 17.3\ \mu\text{sec}.$$

Figure 1 shows the current during the discharge as a function of time, while Fig. 2 gives the discharge channel radius as a function of time.

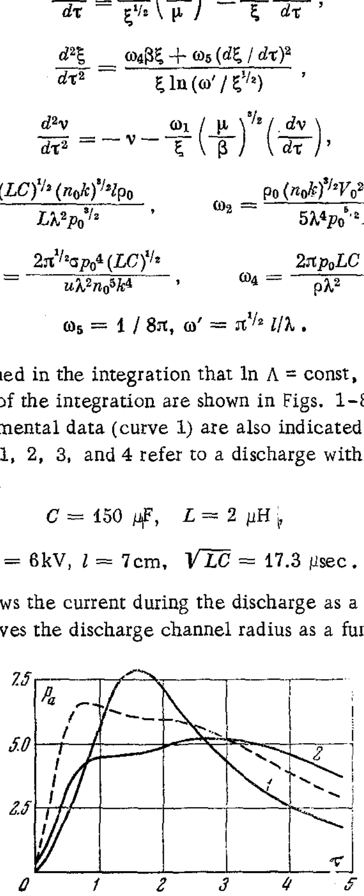

Fig. 4

Figure 3 shows the particle density in the discharge channel, and Fig. 4 shows the pressure p_a (in atm) in the compression pulse at a distance of 1 m from the discharge in a direction perpendicular to the discharge channel axis (the experimental data were obtained by N. A. Roi. The discharges can be described by the following formula [9] (see also [4] for further details)

$$p_a = \frac{\rho\ddot{W}}{4\pi r} = \frac{\rho\ddot{s}l}{4\pi r}, \qquad (12)$$

where ρ is the density of water, r is the distance from the source of sound (discharge) to the point of observation, and W is the volume of the sound source (volume of the discharge channel).

Under our conditions, calculations show that the discharge is aperiodic, and this was confirmed by experiment. The calculated tem-

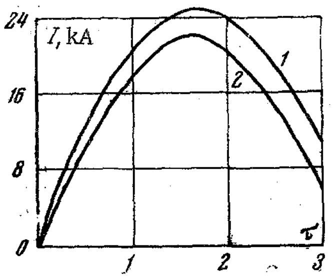

Fig. 5

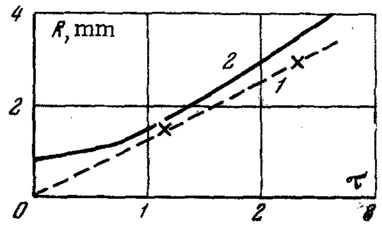

Fig. 6

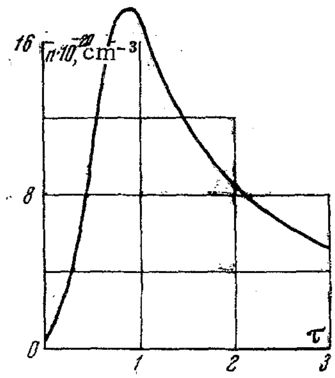

Fig. 7

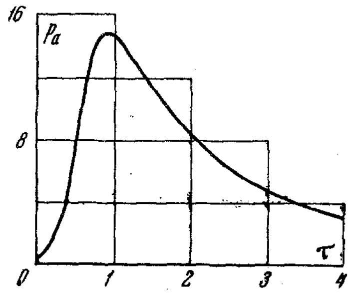

Fig. 8

perature was found to be a very slowly varying function in the range 9500–10 500° K.

Figures 5, 6, 7, and 8 show the corresponding results for the discharge described in [8] with the parameters $C = 2.7\ \mu F$, $L = 7\ \mu H$, $V_0 = 40$ kV, $l = 1.5$ cm, $(LC)^{1/2} = 4.3\ \mu sec$.

In this case, it turns out that the discharge is periodic, which is also in agreement with experiment. The corresponding discharge temperature has a maximum of ≈22 000° K.

In both cases it is clear that the mass of the gas in the channel increases during the discharge process.

At a temperature of T ≈ 10 000° K, the maximum of the black-body emission lies near the visible part of the spectrum, and the water becomes transparent for such wavelengths, i. e., only a part of the flux σT^4 is used for evaporation. In Figs. 1–4 the dashed line shows the result of calculations based on the assumption that the amount of radiation used up through evaporation was $q' = 0.1\ \sigma T^4$, i. e., light with wavelengths below 2000 Å is absorbed for T = 10 000° K. Calculations then show that the temperature in the channel will reach values of the order of 18 000° K quite rapidly, and will vary slowly within the range 18 000–19 000° K for the times indicated in the figures.

The author is grateful to Yu. P. Raizer for advice and attention, and K. A. Naugol'nykh and N. A. Roi for useful discussions.

REFERENCES

1. A. I. Ioffe, K. A. Naugol'nykh, and N. A. Roi, "Initial stage of electrical discharge in water," PMTF, no. 4, 1964.

2. E. Martin, "Experimental investigation of high-energy arc plasma," J. Appl. Phys., vol. 31, 255, 1960.

3. L. Spitzer, Physics of Fully Ionized Gases [Russian translation], Izd. Mir, 1965.

4. K. A. Naugol'nykh and N. A. Roi, "Relation between hydrodynamic and electrical characteristics of discharge in a liquid," Dokl. AN SSSR, vol. 168, 556, 1966.

5. Ya. I. Frenkel, Kinetic Theory of Liquids [in Russian], Izd. GTTL, 1945.

6. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Fizmatgiz, 1963.

7. J. Kaye and T. Laby, Tables of Physical and Chemical Constants [Russian translation], IIL, 1963.

8. Yu. A. Skvortsov, V. S. Komel'kov, and N. M. Kuznetsov, "Expansion off a spark channel in a liquid," Zh. Tekhn. fiz., vol. 30, 1165, 1960.

9. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media, Pergamon Press, 1954.

10. S. I. Anisimov, A. M. Bonch-Bruevich, M. A. El'yashevich, et al., "Effect of high-intensity light beams on metals," Zh. tekhn. fiz., vol. 36, 1213, 1966.

30 May 1966

Acoustics Institute, AS USSR, Moscow